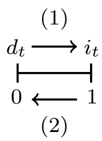
A1. Measurement of Interest

ACCUMULATION FUNCTION

- Notation: $a(t)$
- accumulated value at time t of \$1 invested at time 0
- $a(0)$ is always 1 by definition

EFFECTIVE RATE OF INTEREST & DISCOUNT

- $i_t = \dfrac{a(t) - a(t-1)}{a(t-1)}$ and $d_t = \dfrac{a(t) - a(t-1)}{a(t)}$
- i_t is the amount of interest earned in period t divided by the amount invested at the beginning of the period
- d_t is the amount of interest earned in period t divided by the amount invested at the end of the period
- 

$\dfrac{d_t}{1-d_t} = i_t$ (1)
$\dfrac{i_t}{1+i_t} = d_t$ (2)

DISCOUNT FUNCTION

- Notation: $a^{-1}(t) = 1/a(t)$
- present value at time 0 of \$1 paid at time t

DISCOUNT FACTOR

- Notation: v_t
- discount factor for period t
- discounts money from time t to $t-1$
- $v_t = 1 - d_t = \dfrac{1}{1+i_t}$

COMPOUND INTEREST

- $a(t) = (1+i)^t$
- the effective interest rate is constant each period
- the amount of interest varies each period

SIMPLE INTEREST

- $a(t) = 1 + it$
- the effective interest rate varies for each period
- the amount of interest is constant each period

EQUATIONS OF VALUE

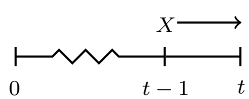
time value of inflows = time value of outflows

NOMINAL RATES OF INTEREST & DISCOUNT

- $\frac{i^{(m)}}{m}$ is the effective rate of interest per m-th of a year
- $\frac{d^{(n)}}{n}$ is the effective rate of discount per n-th of a year
- $\left(1 + \frac{i^{(m)}}{m}\right)^m = \left(1 - \frac{d^{(n)}}{n}\right)^{-n}$ both are accumulation factors for 1 year

SUMMARY OF WAYS TO ACCUMULATE

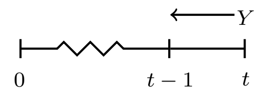
- to accumulate from time $t-1$ to t



(1) $\times \dfrac{a(t)}{a(t-1)}$ (2) $\times (1+i_t)$ (3) $\div (1-d_t)$ (4) $\div v_t$

SUMMARY OF WAYS TO DISCOUNT

- to discount from time t to $t-1$



(1) $\times \dfrac{a(t-1)}{a(t)}$ (2) $\div (1+i_t)$ (3) $\times (1-d_t)$ (4) $\times v_t$

FORCE OF INTEREST

- $\delta_t = \dfrac{a'(t)}{a(t)}$
- If $\delta_t^* = k\delta_t$, then $a^*(t) = [a(t)]^k$.
- to accumulate from time t_1 to t_2 multiply by:
  $$a(t_1, t_2) = \exp\left(\int_{t_1}^{t_2} \delta_t \, dt\right)$$
- to discount from time t_2 to t_1 multiply by:
  $$a^{-1}(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \delta_t \, dt\right)$$
- For compound interest: $\delta_t = \dfrac{(1+i)^t \ln(1+i)}{(1+i)^t} = \ln(1+i)$
- So, under compound interest, force of interest is constant (does not vary by t): $\delta = \ln(1+i)$ or $i = e^\delta - 1$.