A.1 Examples and Exercises

A.1.1 - Interest Accumulation – Part 1

Slide 5/9 (Example 1):

An investment of 100 is made into a fund at time $t = 0$. The fund develops the following balances over the next 4 years:

t	$A(t)$
0	100.00
1	105.00
2	110.25
3	115.76
4	121.55

Find the amount of interest earned during the third year.

Slide 7/9 (Example 2):

Given $A(t) = t^2 + 3t + 5$, find the corresponding $a(t)$.

Slide 8/9 (Exercise 1):

You are given:

(i) $a(t) = 1 + 0.1t$

(ii) \$1000 is invested at time 0.

Determine the amount of interest earned during year 8.

Slide 9/9 (Exercise 2):

You are given:

(i) $a(t) = bt^2 + c$

(ii) \$100 invested at time 0 will be worth \$200 at time 10.

Find the accumulated value at time 15 of \$500 invested at time 0.

A.1.2 - Interest Accumulation – Part 2

Slide 7/12 (Example 1):

An investment is worth 10 at time 5. If $i_6 = 15\%$, how much is it worth at time 6?

Slide 8/12 (Example 2):

You are given $i_1 = i_2 = i_3 = 8\%$ and $i_4 = 10\%$. Find the accumulated value of 10 in 4 years.

Slide 10/12 (Exercise 1):

Given $A(t) = 10(1.08)^t$. Find i_3 and i_7.

Slide 10/12 (Exercise 2):

i_3 is the effective rate of interest for year 3

$$i_3 = \frac{A(3) - A(2)}{A(2)} = \frac{10(1.08)^3 - 10(1.08)^2}{10(1.08)^2} = 1.08 - 1 = 0.08$$

i_7 is the effective rate of interest for year 7

$$i_7 = \frac{A(7) - A(6)}{A(6)} = \frac{10(1.08)^7 - 10(1.08)^6}{10(1.08)^6} = 1.08 - 1 = 0.08$$

Hmm. What about year n?

Slide 11/12 (Exercise 3):

Given $A(2) = 100$ and $i_n = 0.03n$, determine $A(5)$.

A.1.3 - Present Value – Part 1

Slide 2/12 (Example 1):

Suppose you need 110 one year from today. If you can invest money at an effective interest rate of 10%, then how much do you need to invest today to have exactly 110 one year from today?

Slide 9/12 (Exercise 1):

Given $a(t) = 1 + 0.08t$, find i_4 and d_4.

Slide 10/12 (Exercise 2):

John can earn the following rates for the next five years:

- Year 1: effective rate of interest = 4%
- Year 2: effective rate of discount = 5%
- Year 3: effective rate of interest = 6%
- Years 4-5: effective rate of discount = 10%

John needs to have 1000 in 5 years. How much should he invest now?

Slide 11/12 (Exercise 3):

John can earn the following rates for the next five years:

- Year 1: effective rate of interest = 4%
- Year 2: effective rate of discount = 5%
- Year 3: effective rate of interest = 6%
- Years 4-5: effective rate of discount = 10%

John invests 1000 today. How much will John have in 5 years?

Slide 12/12 (Exercise 4):

Given $a(t) = 1.08^t$, find v_5.

A.1.4 - Present Value – Part 2

Slide 2/8 (Example 1):

Write i_t in terms of d_t. Hint: first write i_t in terms of $a(t)$.

Slide 4/8 (Example 2):

Given $i_1 = 12\%$ and $d_2 = 12\%$, find d_1 and i_2.

Slide 6/8 (Exercise 1):

Given $i = 12.5\%$, find d.

Slide 7/8 (Exercise 2):

Given $d = 5\%$, find i.

Slide 8/8 (Exercise 3):

You are given:

i. For years one through three, cash flows are discounted at an annual effective rate of 5%.

ii. For years four and five the annual effective discount rate is 6%.

Find the present value of 1000 paid five years from now.

A.1.5 - Compound Interest

Slide 3/12 (Example 1):

100 is borrowed at a compound interest rate of 10% per annum. Find the loan balance after 3 years.

Slide 4/12 (Example 2):

Given compound interest of i per annum, derive $a(t)$.

Slide 5/12 (Example 3):

Given $d = \frac{1}{9}$, find $a(t)$.

Slide 9/12 (Example 4):

Re-work the example from slide 3 using the TVM keys.

100 is borrowed at a compound interest rate of 10% per annum. Find the loan balance after 3 years.

Slide 10/12 (Exercise 1):

\$100 invested for 3 years, at an effective rate of interest i, will earn \$36 of interest.

Find the accumulated value of \$50 invested at the same rate of compound interest i for 5 years.

Slide 11/12 (Exercise 2):

At a certain rate of compound interest:

(i) 1 grows to 3 in x years

(ii) 3 grows to 14 in y years

(iii) 1 grows to 21 in z years

Determine what 5 grows to in $z - x - y$ years.

Slide 12/12 (Exercise 3):

You invested 100 on January 1, 1997. The investment was worth 190 on July 1, 2002. The effective rate of return for the first year was 12%.

Determine the annualized effective rate of return from January 1, 1998 to July 1, 2002.

Slide 2/12 (Example 1):

100 is invested at simple interest of 10% per annum. Find the accumulated value after 3 years.

Slide 6/12 (Example 2):

Given $i = 0.10$ and simple interest is used only for partial years. Find the accumulated value of 100 five and half years from now.

Slide 7/12 (Example 3):

100 is deposited at the beginning of each year for 3 years. If the deposits earn simple interest of 10%, what is the accumulated value at the end of 3 years?

Slide 10/12 (Exercise 1):

A loan is made for five years at a simple interest rate of 12% per annum. What is the equivalent annual effective rate of discount during the fourth year of the loan?

Slide 11/12 (Exercise 2):

At a rate of simple interest i, 10 will accumulate to 15 after x years. What will 20 accumulate, at a simple rate of $2i$, to after $5x$ years?

Slide 12/12 (Exercise 3):

An investment earns 10% compound interest for each complete year and 8% simple interest for each partial year.

A \$100 investment is made on January 1, 2008. What is the accumulated value of the investment on July 1, 2012?

Slide 3/12 (Example 1):

If $i^{(12)} = 6\%$, what is the equivalent annual effective rate of interest i?

Slide 4/12 (Example 2):

Given $i^{(m)}$, what is the equivalent annual effective rate of interest i?

Slide 5/12 (Example 3):

Given $i = 12\%$, find the equivalent rates $i^{(2)}$, $i^{(4)}$ and $i^{(12)}$.

Slide 6/12 (Example 4):

Derive relationship between $i^{(m)}$ and $i^{(n)}$.

Slide 6/12 (Example 5):

Given $i^{(12)} = 0.12$, find the equivalent rate $i^{(4)}$.

Slide 8/12 (Example 6):

The nominal annual rate convertible once every two years is 15%. Find the accumulated value of 1000 in 5 years.

Slide 9/12 (Example 7):

You are given $i^{(12)} = 10\%$. Find the present value of 100 paid 9 years from now. 9 years = 9 years $\times$ 12 months per year = 108 months

Slide 10/12 (Exercise 1):

Convert the following rates using only your calculator (no pen or paper):

Given	Find	Answer
$i = 0.08$	$i^{(12)}$	
$i^{(2)} = 0.06$	i	
$i^{(4)} = 0.10$	$i^{(2)}$	

Slide 11/12 (Exercise 2):

Given $i^{(2)} = 0.08$, find the accumulated value of 500 in 4.5 years.

Slide 12/12 (Exercise 3):

The nominal annual interest rate convertible once every 4 years is 6%. Find the present value of 400 to be paid in 12 years.

A.1.8 - Nominal Annual Rates of Discount

Slide 2/11 (Example 1):

Given $d^{(4)} = 0.08$, find d.

Slide 3/11 (Example 2):

Write d in terms of $d^{(m)}$ and vice versa.

Slide 4/11 (Example 3):

Given $i = 0.12$, find $d^{(2)}$, $d^{(4)}$ and $d^{(12)}$.

Slide 5/11 (Example 4):

Derive relationship between $d^{(m)}$ and $d^{(n)}$.

Slide 5/11 (Example 5):

Given $d^{(2)} = 0.12$, find the equivalent rate $d^{(4)}$.

Slide 7/11 (Example 6):

Given $i^{(12)} = 0.12$, find the equivalent rate $d^{(2)}$.

Slide 9/11 (Exercise 1):

Convert the following rates using only your calculator (no pen or paper):

Given	Find	Answer
$d = 0.04$	$d^{(4)}$	
$d = 0.12$	i	
$d^{(2)} = 0.07$	$i^{(12)}$	
$i^{(4)} = 0.10$	$d^{(2)}$	

Slide 10/11 (Exercise 2):

Given $d^{(2)} = 0.08$, find the accumulated value of 500 in 4.5 years.

Slide 11/11 (Exercise 3):

The nominal annual discount rate convertible once every 4 years is 6%. Find the present value of 400 to be paid in 10 years.

A.1.9 - Force of Interest – Part 1

Slide 5/12 (Example 1):

Given $\delta_t = \frac{a'(t)}{a(t)}$, find $a(n)$ in terms of δ_t.

Slide 7/12 (Example 2):

If $\delta_t = 0.15\sqrt{t}$ and an amount of 5000 is invested at time $t = 1$, what is the accumulated value at time $t = 4$?

Slide 10/12 (Exercise 1):

On July 1, 1984, a person invested \$1000 in a fund for which the force of interest at time t is given by $\delta_t = (3 + 2t)/50$, where t is the number of years since January 1, 1984. Determine the accumulated value of the investment on January 1, 1985.

Slide 11/12 (Exercise 2):

X is deposited into a savings account at time $t = 0$. No other amounts are deposited. The force of interest for the fund is $\delta_t = t/30$. The balance of fund after 10 years is 12,500.

Determine X.

Slide 12/12 (Exercise 3):

\$1000 is deposited into a savings account at time $t = 0$. No other amounts are deposited. The accumulated amount in the account at time t is given by:

$$A(t) = 1000\left(1 + \tfrac{2t}{35}\right)^2$$

Determine the force of interest at time $t = 32.5$.

A.1.10 - Force of Interest – Part 2

Slide 2/11 (Example 1):

Given $\delta_t = \frac{2t}{t^2+1}$, find $a(t)$.

Slide 3/11 (Example 2):

Given $\delta_t = \frac{2t}{t^2+8}$, find $a(t)$.

Slide 4/11 (Example 3):

Given $\delta_t \Rightarrow a(t)$. If $\delta_t^* = k\delta_t$, what is $a^*(t)$?

Slide 4/11 (Example 4):

Given $\delta_t = \frac{4}{1+t}$, find $a(t)$.

Slide 5/11 (Example 5):

Given $a(t) = (1+i)^t$, find δ_t.

Slide 6/11 (Example 6):

If $\delta = 0.05$ and an amount of 5000 is invested at time $t = 1$, what is the accumulated value at time $t = 4$?

Slide 6/11 (Example 7):

If $\delta = 0.08$, find the present value of 1000 to be paid in 4.25 years.

Slide 7/11 (Example 8):

Given $a(t) = 1 + it$, find δ_t.

Slide 8/11 (Exercise 1):

(a) Given $d = 0.02$, find δ.

(b) Given $\delta = 0.06$, find $i^{(12)}$.

Slide 9/11 (Exercise 2):

At a force of interest $\delta = 0.05$, the following payments have the same present value:

(i) X at the end of year 5 plus $2X$ at the end of year 10

(ii) Y at the end of year 14

Calculate Y/X.

Slide 10/11 (Exercise 3):

On 1/1/97, Kelly deposits X into a bank account. The account is credited with simple interest at a rate of 10% per year.

On the same date, Tara deposits X into a different bank account. The account is credited interest using a force of interest:

$$\delta_t = \frac{2t}{t^2 + k}$$

From the end of the 4th year until the end of the 8th year, both accounts earn the same dollar amount of interest.

Calculate k.