

The following problems have been removed since they are not on the 3L syllabus.

3, 22, 32, 38, 43, 44, 45, 47, 51, 54, 58, 66, 67, 69, 73, 92, 94, 102, 103, 108, 111, 115, 122, 123, 125, 128, 129, 130, 131, 135, 136, 145, 150, 156, 160, 165

1. If we assume DeMoivre's mortality law, with $\omega = 105$ and $k = 100,000$, what is ${}_{10|20}q_{25}$?

A. $\frac{1}{8}$

B. $\frac{1}{6}$

C. $\frac{1}{5}$

D. $\frac{1}{4}$

E. $\frac{3}{8}$

2. A subgroup of lives has twice the normal force of mortality, i.e.

$$\mu'(x) = 2\mu(x) \text{ for all } x$$

Which of the following is an expression for q'_x , the probability that a member of this subgroup aged x will die within one year?

- A. $(q_x)^2$ B. $(q_x)^2 - 2q_x$ C. $2q_x - (q_x)^2$ D. $2(q_x)^2 - q_x$ E. $2q_x + (q_x)^2$

4. The force of mortality is:

$$\mu(x) = \frac{1}{100-x}$$

Calculate ${}_{10}p_{50}$

- A. Less than .82
- B. At least .82, but less than .84
- C. At least .84, but less than .86
- D. At least .86, but less than .88
- E. At least .88

5. Which of the following are equivalent to ${}_t p_x$?

A. ${}_t|_u q_x - {}_{t+u} p_x$

B. ${}_{t+u} q_x - {}_t q_x + {}_{t+u} p_x$

C. ${}_t q_x - {}_{t+u} q_x + {}_t p_{x+u}$

D. ${}_t q_x - {}_{t+u} q_x - {}_t p_{x+u}$

E. None of the above

6. Given the following information about a group of lives, what is the complete expectation of life $\overset{\circ}{e}_{39}$?

1. The total number of years lived beyond age 38 is $T_{38} = 95,000$
2. The total expected number of years lived between ages 38 and 39 is $L_{38} = 2,475$
3. The central death rate at age 38 is $m_{38} = .021$
4. The number of survivors to age 38 is $\ell_{38} = 2,500$

- A. Less than 37.0
- B. At least 37.0, but less than 37.2
- C. At least 37.2, but less than 37.4
- D. At least 37.4, but less than 37.6
- E. At least 37.6

7. Which of the following are true?

1. $t|_u q_x = t p_x \cdot u q_{x+t}$

2. $t|_u q_x = \frac{\ell_{x+t+u} - \ell_{x+t}}{\ell_x}$

3. $t|_u q_x = t p_x - {}_{t+u} p_x$

A. 1

B. 2

C. 3

D. 1, 2

E. 1, 3

8. Given that a life aged 50 will live to age 60, what is the probability p that he will die between ages 70 and 80?

Age	ℓ_x
50	89,509
60	81,881
70	66,162
80	39,144

- A. Less than .310
- B. At least .310, but less than .315
- C. At least .315, but less than .320
- D. At least .320, but less than .325
- E. At least .325

9. If $\ell_x = 1000 \cdot (1 - \frac{x}{100})$ for $0 \leq x \leq 100$, calculate the curtate expectation of life, e_{90} .
- A. Less than 4.2
 - B. At least 4.2, but less than 4.4
 - C. At least 4.4, but less than 4.6
 - D. At least 4.6, but less than 4.8
 - E. At least 4.8

10. You are given the following information:

1. Mortality follows de Moivre's law, $l_x = l_0 \cdot (\omega - x)$ for $0 \leq x \leq \omega$.

2. ${}^{\circ}e_{20} = 45$

Calculate the variance of the future lifetime of a person aged 20, $\text{Var}[T(20)]$, to the nearest integer.

A. 108

B. 275

C. 350

D. 675

E. 700

11. Which of the following represents an expression for the complete expectation of life?

1. ${}^{\circ}e_x = e_x - \frac{1}{2}$ (assuming uniform distribution of deaths)

2. ${}^{\circ}e_x = \int_0^{\infty} t \cdot {}_t p_x \cdot \mu(x+t) \cdot dt$

3. ${}^{\circ}e_x = \frac{\int_0^{\infty} \ell_{x+t} \cdot dt}{\ell_x}$

A. 1 only

B. 2 only

C. 1 and 2 only

D. 2 and 3 only

E. 1, 2 and 3

12. You are given the following information:

1. The force of mortality is constant for all ages.
2. The force of mortality equals the force of interest.
3. $\bar{a}_{\overline{20}|} = 1.4 \cdot \bar{a}_{\overline{10}|}$

Calculate the probability that (20) will die between the ages of 40 and 50.

- A. Less than .085
- B. At least .085, but less than .090
- C. At least .090, but less than .095
- D. At least .095, but less than .10
- E. At least .10

13. If $\mu(x) = 0.0012$ for $15 \leq x \leq 20$, find ${}_2|_2q_{15}$.

- A. Less than .0020
- B. At least .0020, but less than .0023
- C. At least .0023, but less than .0026
- D. At least .0026, but less than .0029
- E. At least .0029

14. State X has a constant number of licensed drivers. Each year 1000 new drivers are licensed at age 15 and all drivers relinquish their licenses at age 65. Death is the only reason for losing one's license. Assuming a uniform distribution of deaths and given $\ell_x = 1000 - 10 \cdot x$ for $0 \leq x \leq 100$, find the total number of licensed drivers in State X.
- A. Less than 30,000
 - B. At least 30,000, but less than 35,000
 - C. At least 35,000, but less than 40,000
 - D. At least 40,000, but less than 45,000
 - E. At least 45,000

15. You are given that the force of mortality, $\mu(30 + t)$ for $0 \leq t \leq 1$, changes to $\mu(30 + t) - c$ where c is a positive constant. You are also given that p_{30} is 0.95. Determine c such that the probability that (30) will die within one year will be reduced by 25%.
- A. Less than .014
 - B. At least .014, but less than .015
 - C. At least .015, but less than .016
 - D. At least .016, but less than .017
 - E. At least .017

16. You are given $s(x) = e^{-\frac{x^3}{12}}$, for $x \geq 0$.

Determine $\mu(x)$.

A. $-\frac{x^2}{4}$

B. $1 - \frac{x^2}{4}$

C. $\frac{x^2}{4}$

D. $\frac{x^2}{4} e^{-\frac{x^2}{12}}$

E. $-\frac{x^3}{12}$

17. You are given a survivorship group with 1000 initial members at age zero and the survival function of $s(x) = 1 - \frac{x}{125}$ for $0 \leq x \leq 125$, where x is age.

Calculate T_{50} , the total number of years lived beyond age 50 by the survivorship group.

- A. Less than 20,000
- B. At least 20,000, but less than 30,000
- C. At least 30,000, but less than 40,000
- D. At least 40,000, but less than 50,000
- E. At least 50,000

18. A mortality table for a subset of the population with better than average health is constructed by dividing the force of mortality in the standard table by 2. The probability of an 80-year-old dying within the next year is defined in the standard table as q_{80} and in the revised table it is defined q'_{80} . In the standard table $q_{80} = 0.30$. Determine the value of q'_{80} in the revised table.
- A. Less than 0.150
 - B. At least 0.150, but less than 0.155
 - C. At least 0.155, but less than 0.160
 - D. At least 0.160, but less than 0.165
 - E. At least 0.165

19. You are given a survival function $s(x) = 1 - .01x$ for $0 \leq x \leq 100$.

Determine the median future lifetime of a life aged 10.

- A. Less than 42
- B. At least 42, but less than 44
- C. At least 44, but less than 46
- D. At least 46, but less than 48
- E. At least 48

20. You are given the following mortality table:

Age(x)	q_x	ℓ_x	d_x
20		30,000	1,200
21			
22		27,350	
23	0.0700		
24	0.0790	23,900	

Determine the probability that a life aged 21 will die within two years.

- A. Less than 0.0960
- B. At least 0.0960, but less than 0.1010
- C. At least 0.1010, but less than 0.1060
- D. At least 0.1060, but less than 0.1110
- E. At least 0.1110

21. You are given:

$$s(x) = 1 - \frac{x}{100} \text{ for } 0 \leq x \leq 100$$

Determine $\text{Var}[T(20)]$, where $T(20)$ is the future-lifetime-of-(20) random variable.

- A. Less than 600
- B. At least 600, but less than 800
- C. At least 800, but less than 1,000
- D. At least 1,000, but less than 1,200
- E. At least 1,200

23. You are given that hens lay an average of 30 eggs each month until death and you are given the survival function for hens is:

$$s(m) = 1 - \frac{m}{72}, \text{ for } 0 \leq m < 72 \text{ where } m \text{ is in months}$$

Further assume that 100 hens have survived to age 12 months.

Calculate the expected total number of eggs to be laid by these 100 hens in their remaining lifetime.

- A. Less than 33,000
- B. At least 33,000, but less than 57,000
- C. At least 57,000, but less than 81,000
- D. At least 81,000, but less than 105,000
- E. At least 105,000

24. You are given that a survival function follows Weibull's law with the following information:

- $s(x) = e^{-ux^{n+1}}$
- $\mu(x) = kx^n$
- $k = 0.0165$
- $u = 0.015$
- $n = 0.10$

Calculate $\frac{d}{dt} {}_tq_x$ for an insured aged 25, with $t = 10$.

- A. Less than 0.0165
- B. At least 0.0165, but less than 0.0175
- C. At least 0.0175, but less than 0.0185
- D. At least 0.0185, but less than 0.0195
- E. At least 0.0195

25. You are given:

- $\ell_0 = 1,000$
- $\mu(x) = \frac{1}{70-x}$, for $0 \leq x < 70$

Determine the expected density of deaths when $x = 30$.

- A. Less than 14.0
- B. At least 14.0, but less than 14.5
- C. At least 14.5, but less than 15.0
- D. At least 15.0, but less than 15.5
- E. At least 15.5

26. You are given the survival function $s(x) = e^{-0.05x}$ for $x > 0$.

Determine the central-death-rate, m_x , at age 30.

- A. Less than 0.045
- B. At least 0.045, but less than 0.047
- C. At least 0.047, but less than 0.049
- D. At least 0.049, but less than 0.051
- E. At least 0.051

27. You are given a survivorship group with 500 initial members at age 0. You are also given that:

$$\mu(x) = \frac{1}{100-x}, \text{ for } 0 \leq x < 100, \text{ where } x \text{ is the age.}$$

Determine T_{25} , the total number of years lived beyond age 25 by the survivorship group.

- A. Less than 13,900
- B. At least 13,900, but less than 14,100
- C. At least 14,100, but less than 14,300
- D. At least 14,300, but less than 14,500
- E. At least 14,500

28. A life table for severely disabled lives is created by modifying an existing life table by doubling the force of mortality at all ages.

In the original table, $q_{75} = 0.12$.

Calculate q_{75} in the modified table.

- A. Less than 0.21
- B. At least 0.21, but less than 0.23
- C. At least 0.23, but less than 0.25
- D. At least 0.25, but less than 0.27
- E. At least 0.27

29. A population of 20,000 lives has two subpopulations. The first subpopulation has 10,000 lives, all age 30, with their mortality described by DeMoivre survival function where $\omega = 100$. The other subpopulation has 10,000 lives, all age 40, with a DeMoivre survival function where $\omega = 90$.

Determine the expected number of people from this population of 20,000 who will die between the ages of 50 and 60.

- A. Less than 3,300
- B. At least 3,300, but less than 3,500
- C. At least 3,500, but less than 3,700
- D. At least 3,700, but less than 3,900
- E. At least 3,900

30. At birth, infants are subject to a decreasing force of mortality during the early months of life. Assume a newborn infant is subject to a force of mortality given by:

$$\mu(x) = \frac{1}{10+x} \quad , \text{ for } x \geq 0, \text{ where } x \text{ is expressed in months.}$$

Calculate the probability that a newborn infant will survive 5 months and die in the ensuing 15 months.

- A. Less than 0.15
- B. At least 0.15, but less than 0.20
- C. At least 0.20, but less than 0.25
- D. At least 0.25, but less than 0.30
- E. At least 0.30

31. You are given 2 cohorts of 10,000 lives each, all aged 0. One cohort experiences a constant force of mortality $\mu(x) = 0.05$. The other cohort experiences a force of mortality given by $\mu(y) = \frac{1}{100-y}$ for $0 \leq y < 100$.

Determine the difference in the total number of years lived beyond aged 80 between the cohorts.

- A. Less than 15,000
- B. At least 15,000, but less than 16,000
- C. At least 16,000, but less than 17,000
- D. At least 17,000, but less than 18,000
- E. At least 18,000

33. Given that the force of mortality $\mu(x) = 2x$, determine the cumulative distribution function for the random variable time until death, $F(x)$, the density function for that random variable, $f(x)$, and the survival function $s(x)$.

- A. $F(x) = 1 - e^{x^2}$ $f(x) = 2xe^{-x^2}$ $s(x) = e^{-x^2}$
- B. $F(x) = 1 - e^{-x^2}$ $f(x) = 2e^{-x^2}$ $s(x) = e^{x^2}$
- C. $F(x) = 1 - 2x$ $f(x) = 2e^{-x^2}$ $s(x) = e^{x^2}$
- D. $F(x) = 1 - e^{x^2}$ $f(x) = xe^{-x^2}$ $s(x) = e^{2x}$
- E. $F(x) = 1 - e^{-x^2}$ $f(x) = 2xe^{-x^2}$ $s(x) = e^{-x^2}$

34. You are given a life, aged 30, subject to a force of mortality given by:

$$\mu(x) = 0.02 \cdot x^{0.5}, \text{ for } 20 \leq x \leq 50.$$

Determine the probability this life will survive 5 years and die during the following year.

- A. Less than 0.044
- B. At least 0.044, but less than 0.052
- C. At least 0.052, but less than 0.060
- D. At least 0.060, but less than 0.068
- E. At least 0.068

35. You are given the following information about two lives:

Life	Future Lifetime Random Variables
x	Constant force of mortality $\mu(x) = 0.10$
y	Constant force of mortality $\mu(y) = 0.20$

Determine the ratio of (x 's expected future lifetime between ages x and $x + 10$) to (y 's expected future lifetime between ages y and $y + 10$).

- A. Less than 1.00
- B. At least 1.00, but less than 1.25
- C. At least 1.25, but less than 1.50
- D. At least 1.50, but less than 1.75
- E. At least 1.75

36. For a life aged 50, the curtate-expectation of life $e_{50} = 20$. For that same life, you are also given that $p_{50} = 0.97$.

Determine e_{51} .

- A. Less than 18.75
- B. At least 18.75, but less than 19.00
- C. At least 19.00, but less than 19.25
- D. At least 19.25, but less than 19.50
- E. At least 19.50

37. Calculate the probability that a 40-year-old will survive to age 42 if mortality follows Weibull's law: $\mu(x) = kx^n$, with $k = \frac{1}{100}$ and $n = 1$.
- A. Less than 0.20
 - B. At least 0.20, but less than 0.30
 - C. At least 0.30, but less than 0.40
 - D. At least 0.40, but less than 0.50
 - E. At least 0.50

39. Given the following portion of a life table:

x	ℓ_x	d_x	p_x	q_x
0	1,000		0.875	
1				
2	750			0.25
3				
4				
5	200	120		
6				
7		20		1.00

Determine the value of $p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot q_6$.

- A. Less than 0.055
- B. At least 0.055, but less than 0.065
- C. At least 0.065, but less than 0.075
- D. At least 0.075
- E. The answer cannot be determined from the given information.

40. If $\mu(x) = \frac{1}{2(100-x)}$, calculate ${}_{40}p_{25}$.

- A. Less than 0.64
- B. At least 0.64, but less than 0.66
- C. At least 0.66, but less than 0.68
- D. At least 0.68, but less than 0.70
- E. At least 0.70

41. If $\ell_x = 100(k - 0.5x)^{\frac{2}{3}}$ and $\mu(50) = \frac{1}{48}$, find k .

A. 40

B. 41

C. 42

D. 43

E. 44

42. Which of the following equations define valid mortality functions?

1. $\mu(x) = (1+x)^{-3}, x \geq 0$

2. $\mu(x) = .05(1.01)^x, x \geq 0$

3. $f_X(x) = e^{-\frac{x}{2}}, x \geq 0$

A. 1

B. 2

C. 1, 2

D. 1, 3

E. 2, 3

46. You are given $\mu(x) = \frac{2x}{10,000-x^2}$ for $0 \leq x < 100$. Determine q_x .

A. $\frac{2x+1}{10,000-x^2}$

B. $\frac{4x+2}{10,000-x^2}$

C. $\frac{6x+3}{10,000-x^2}$

D. $\frac{2x+1}{29,999-3x^2-3x}$

E. $\frac{6x+3}{29,999-3x^2-3x}$

48. You are given:

1. $q_{60} = 0.020$

2. $q_{61} = 0.022$

3. Deaths are uniformly distributed over each year of age.

Calculate ${}^{\circ}e_{60:\overline{1.5}|}$.

A. 1.447

B. 1.457

C. 1.467

D. 1.477

E. 1.487

49. You are given:

- $\delta = 0$
- $\int_0^\infty t \cdot {}_t p_x dt = g$
- $\text{Var} [\bar{a}_{\overline{T}|}] = h$, where T is the future lifetime random variable for (x) .

Express \ddot{e}_x in terms of g and h .

A. $h - g$

B. $\sqrt{h - g}$

C. $\sqrt{g - h}$

D. $\sqrt{2g - h}$

E. $\sqrt{2h - g}$

50. You are given:

1. $\ell_{70} = 1055$
2. $a(70) = 0.450$, the average number of years of life after age 70 for those who die between ages 70 and 71.
3. $m_{70} = 0.100$, the central death rate for age 70.

Calculate ℓ_{71} .

A. 940

B. 955

C. 967

D. 973

E. 976

52. You are given:

1. $d_x = k, x = 0, 1, 2, \dots, \omega - 1$

2. ${}^{\circ}e_{20:\overline{20}|} = 18$

3. Deaths are uniformly distributed over each year of age.

Calculate ${}_{30|10}q_{30}$.

A. 0.111

B. 0.125

C. 0.143

D. 0.167

E. 0.200

53. You are given:

1. $\ell_x = (100 - x)^{0.5}, 0 \leq x \leq 100$

2. ${}^{\circ}e_{36:\overline{28}|} = 24.67$

Calculate $\int_0^{28} t \cdot {}_tp_{36} \mu(36 + t) dt$.

A. 3.67

B. 5.00

C. 11.33

D. 19.67

E. 24.67

55. You are given:

1. $0.5p_x \mu(x + 0.5) = \frac{12}{49}$

2. $q_x < p_x$

Assuming uniform distributions of deaths, calculate q_x .

A. $\frac{6}{25}$

B. $\frac{12}{49}$

C. $\frac{1}{4}$

D. $\frac{12}{47}$

E. $\frac{6}{23}$

56. Which of the following functions can serve as a force of mortality?

1. Bc^x $B > 0, 0 < c < 1, x \geq 0$
2. $B(x+1)^{-0.5}$ $B > 0, x \geq 0$
3. $k(x+1)^n$ $k > 0, n > 0, x \geq 0$

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

57. Assume mortality follows DeMoivre's law for $0 \leq x < \omega$. The median future lifetime of (x) is denoted by $m(x)$. Which of the following are equal to $\mu(x)$ for $1 \leq x \leq \omega - 1$?

1. $\frac{q_{x-1}}{p_{x-1}}$

2. $\frac{1}{2m(x)}$

3. $\frac{m_x}{1 + 0.5m_x}$

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

59. You are given:

- Deaths are uniformly distributed over each year of age.

x	ℓ_x
35	100
36	99
37	96
38	92
39	87

Which of the following are true?

1. ${}_1|_2q_{36} = 0.091$ 2. $m_{37} = 0.043$ 3. ${}_{\frac{1}{3}}q_{38.5} = 0.021$

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

60. You are given:

1. Deaths are uniformly distributed over each year of age.

2. $\mu(45.5) = 0.5$

Calculate ${}^{\circ}e_{45:\overline{1}|}$.

A. 0.4

B. 0.5

C. 0.6

D. 0.7

E. 0.8

61. You are given:

1. $q_{70} = 0.040$

2. $q_{71} = 0.044$

3. Deaths are uniformly distributed over each year of age.

Calculate ${}^{\circ}e_{70:\overline{1.5}|}$.

A. 1.435

B. 1.445

C. 1.455

D. 1.465

E. 1.475

62. You are given:

1. $q_x = 0.25$

2. $\ln(\frac{4}{3}) = 0.2877$

Based on the constant force of mortality assumption, the force of mortality is $\mu(x+s)$, $0 < s < 1$.

Based on the uniform distribution of deaths assumption, the force of mortality is $\mu^*(x+s)$, $0 < s < 1$.

Calculate the smallest s such that $\mu^*(x+s) \geq \mu(x+s)$.

A. 0.4523

B. 0.4758

C. 0.5001

D. 0.5242

E. 0.5477

63. You are given:

1. $\mu(x) = A + e^x$ for $x \geq 0$

2. ${}_{0.5}p_0 = 0.50$

Calculate A .

A. -0.26

B. -0.09

C. 0.00

D. 0.09

E. 0.26

64. Assume mortality follows DeMoivre's law, for $0 \leq x < \omega$. Which of the following expressions equal $\mu(x)$?

1. $\frac{1}{2} \frac{e_x}{e_x}$

2. ${}_nq_x, 0 \leq n \leq \omega - x - 1$

3. $\frac{m_x}{1 + 0.5m_x}, x \leq \omega - 1$

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

65. Which of the following can serve as survival functions for $x \geq 0$?

1. $s(x) = \exp(x - 0.7(2^x - 1))$

2. $s(x) = \frac{1}{(1+x)^2}$

3. $s(x) = \exp(-x^2)$

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

68. A survival function, $s(x)$, is defined as follows:

$$s(x) = \left(1 - \frac{x}{\omega}\right)^r, \text{ for } 0 \leq x < \omega, x > 0$$

For age y , $0 \leq y < \omega$, you are given:

- $\mu(y) = 0.1$
- ${}^{\circ}e_y = 8.75$

Calculate r .

A. 1

B. 3

C. 5

D. 7

E. 9

70. You are given:

- Deaths are uniformly distributed over each year of age.
- $0.75p_x = 0.25$

Which of the following are true?

1. $0.25q_{x+0.5} = 0.5$
2. $0.5q_x = 0.5$
3. $\mu(x + 0.5) = 0.5$

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

71. You are given:

1. $\mu^G(x)$ denotes the force of mortality under Gompertz's law at age x where $B = 0.05$ and $c = 10^{0.04}$.
2. $\mu^W(x)$ denotes the force of mortality under Weibull's law at age x where $k = 0.1$ and $n > 0$.
3. $\mu^G(50) = \mu^W(50)$

Calculate n .

A. 0.5

B. 1.0

C. 1.5

D. 2.0

E. 2.5

72. You are given:

$$s(x) = \left(1 - \frac{x}{\omega}\right)^\alpha, \text{ for } 0 \leq x < \omega, \text{ where } \alpha \text{ is a positive constant.}$$

Calculate $\mu(x) \cdot \overset{\circ}{e}_x$.

A. $\frac{\alpha}{\alpha + 1}$

B. $\frac{\alpha\omega}{\alpha + 1}$

C. $\frac{\alpha^2}{\alpha + 1}$

D. $\frac{\alpha^2}{\omega - x}$

E. $\frac{\alpha(\omega - x)}{(\alpha + 1)\omega}$

74. You are given:

1. $\mu(35+t) = \mu$, $0 \leq t \leq 1$
2. $p_{35} = 0.985$
3. $\mu'(35+t)$ is the force of mortality for (35) subject to an additional hazard, $0 \leq t \leq 1$
4. $\mu'(35+t) = \mu + c$, $0 \leq t \leq 0.5$
5. The additional force of mortality decreases uniformly from c to 0 between ages 35.5 and age 36.

Determine the probability that (35) subject to the additional hazard will not survive to age 36.

- A. $0.015e^{-0.25c}$
- B. $0.015e^{0.25c}$
- C. $1 - 0.985e^{-c}$
- D. $1 - 0.985e^{-0.5c}$
- E. $1 - 0.985e^{-0.75c}$

75. For the current type of refrigerator, you are given:

- $s(x) = 1 - \frac{x}{\omega}$, for $0 \leq x \leq \omega$
- ${}^{\circ}e_0 = 20$

For a proposed new type, with the same ω , the new survival function is:

$$s^*(x) = \begin{cases} 1, & 0 \leq x \leq 5 \\ \frac{\omega - x}{\omega - 5}, & 5 < x \leq \omega \end{cases}$$

Calculate the increase in life expectancy at time 0.

A. 2.25

B. 2.50

C. 2.75

D. 3.00

E. 3.25

76. You are given:

$$\mu(x) = \frac{4}{100 - x}, \quad 0 \leq x < 100$$

Calculate the average number of years lived between ages 40 and 60 by those currently age 30.

A. 2.6

B. 4.8

C. 5.6

D. 6.5

E. 9.1

77. You are given:

1. $s(x) = \frac{(k^3 - x)^{1/3}}{k}, 0 \leq x \leq k^3, k > 0$

2. ${}^{\circ}e_{40} = 2 {}^{\circ}e_{80}$

Calculate ${}^{\circ}e_{60}$

A. 40

B. 45

C. 50

D. 55

E. 60

78. You are given the survival function $s(x) = \frac{\sqrt{100-x}}{10}$ for $0 \leq x \leq 100$.

Calculate $F(75)$, $f(75)$, and $\mu(75)$.

- A. $F(75) = 0.2$ $f(75) = 0.02$ $\mu(75) = 0.04$
- B. $F(75) = 0.5$ $f(75) = 0.01$ $\mu(75) = 0.02$
- C. $F(75) = 0.5$ $f(75) = 0.01$ $\mu(75) = 0.04$
- D. $F(75) = 0.2$ $f(75) = 0.02$ $\mu(75) = 0.02$
- E. $F(75) = 0.5$ $f(75) = 0.02$ $\mu(75) = 0.04$

79. You are given:

- ${}_1|q_{x+1} = 0.095$
- ${}_2|q_{x+1} = 0.171$
- $q_{x+3} = 0.200$

Calculate $q_{x+1} + q_{x+2}$.

A. 0.15

B. 0.20

C. 0.25

D. 0.27

E. 0.30

80. You are given:

x	ℓ_x
96	180
97	130
98	73
99	31
100	0

Define K to be the curtate future lifetime of (96). Calculate $\text{Var}(K)$.

A. 0.39

B. 0.53

C. 0.91

D. 1.11

E. 1.50

81. Simplify $\frac{d}{dx}(qx)$.

A. $p_x \mu(x+1)$

B. $\mu(x) - \mu(x+1)$

C. $-(\mu(x) - \mu(x+1))$

D. $p_x (\mu(x) - \mu(x+1))$

E. $-p_x (\mu(x) - \mu(x+1))$

82. You are given:

1. $\ell_x = \omega^3 - x^3$ for $0 \leq x \leq \omega$

2. $E[T(0)] = (3/4)\omega$

Calculate $\text{Var}[T(0)]$.

A. $\frac{3}{125}\omega^2$

B. $\frac{3}{80}\omega^2$

C. $\frac{3}{20}\omega^2$

D. $\frac{9}{20}\omega^2$

E. $\frac{9}{16}\omega^2$

83. You are given $\mu(x) = kx$ for all $x > 0$ and $_{10}p_{35} = 0.81$.

Calculate $_{20}p_{40}$.

A. 0.36

B. 0.41

C. 0.45

D. 0.59

E. 0.66

84. You are given ${}_t|q_x = 0.10$, for $t = 0, 1, \dots, 9$.

Calculate ${}_2p_{x+5}$.

A. 0.40

B. 0.60

C. 0.72

D. 0.80

E. 0.81

85. You are given:

1. Mortality follows DeMoivre's Law

2. ${}^{\circ}e_{30} = 30$.

Calculate q_{30} .

A. $\frac{1}{30}$

B. $\frac{1}{60}$

C. $\frac{1}{61}$

D. $\frac{1}{62}$

E. $\frac{1}{70}$

86. A mortality table has a force of mortality $\mu(x+t)$ and mortality rate q_x . A second mortality table has a force of mortality $\mu^*(x+t)$ and mortality rate q_x^* . You are given $\mu^*(x+t) = 0.5\mu(x+t)$ for $0 \leq t \leq 1$.

Calculate q_x^* .

A. $1 - \sqrt{1 - q_x}$

B. $\sqrt{q_x}$

C. $0.5q_x$

D. $(q_x)^2$

E. $q_x - (q_x)^2$

87. You are given $s(x) = \frac{1}{1+x}$.

Determine the median future lifetime of (y) .

A. $y + 1$

B. y

C. 1

D. $\frac{1}{y}$

E. $\frac{1}{1+y}$

88. You are given:

1. $\hat{\mu}(x+t) = \mu(x+t) - k, 0 \leq t \leq 1$

2. $\hat{q}_x = 0$, where \hat{q}_x is based on the force of mortality $\hat{\mu}(x+t)$.

Determine k .

A. $-\ln p_x$

B. $\ln p_x$

C. $-\ln q_x$

D. $\ln q_x$

E. q_x

89. You are given $F(x) = 1 - \frac{1}{x+1}$ for $x \geq 0$.

Which of the following are true?

1. ${}_x p_0 = \frac{1}{x+1}$

2. $\mu(49) = 0.02$

3. ${}_{10}p_{39} = 0.80$

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

90. You are given:

1. Mortality follows DeMoivre's Law.

2. $\text{Var}[T(15)] = 675$.

Calculate ${}^{\circ}e_{25}$.

A. 37.5

B. 40.0

C. 42.5

D. 45.0

E. 47.5

91. You are given:

1. $q_x = 0.04$

2. $\mu(x+t) = 0.04 + 0.001644t, \ 0 \leq t \leq 1$

3. $\mu(y+t) = 0.08 + 0.003288t, \ 0 \leq t \leq 1$

Calculate q_y .

A. 0.0784

B. 0.0792

C. 0.0800

D. 0.0808

E. 0.0816

93. You are given:

1. $R = 1 - e^{-\int_0^1 \mu(x+t) dt}$
2. $S = 1 - e^{-\int_0^1 (\mu_{x+t}-k) dt}$
3. k is a positive constant.

Determine an expression for k such that $S = \frac{2}{3}R$.

- A. $\ln \left[(1 - p_x) / (1 - \frac{2}{3}q_x) \right]$
- B. $\ln \left[(1 - \frac{2}{3}q_x) / (1 - p_x) \right]$
- C. $\ln \left[(1 - \frac{2}{3}p_x) / (1 - p_x) \right]$
- D. $\ln \left[(1 - q_x) / (1 - \frac{2}{3}q_x) \right]$
- E. $\ln \left[(1 - \frac{2}{3}q_x) / (1 - q_x) \right]$

95. You are given:

1. $s(x) = \frac{\sqrt{k^2 - x}}{k}$, $0 \leq x \leq k^2$, $k > 0$

2. ${}^{\circ}e_{40} = 2 {}^{\circ}e_{80}$

Calculate ${}^{\circ}e_{60}$.

A. 10

B. 20

C. 30

D. 40

E. 50

96. You are given:

$$\mu(x) = \sqrt{\frac{1}{80-x}}, \quad 0 \leq x < 80$$

Calculate the median future lifetime of (20).

A. 5.25

B. 6.08

C. 8.52

D. 26.08

E. 30.00

97. You are given:

1. ${}_tp_x = (0.8)^t, t \geq 0$

2. $\ell_{x+2} = 6.4$

Calculate T_{x+1} .

A. 4.5

B. 7.2

C. 28.7

D. 35.9

E. 44.8

98. You are given:

1. T is the random variable for the future lifetime of (x) .
2. The p.d.f. of T is $f_T(t) = 2e^{-2t}$, $t \geq 0$.

Calculate ${}^{\circ}e_x$.

A. 0.5

B. 2.0

C. 10.0

D. 20.0

E. 40.0

99. You are given:

1. T is the random variable for the future lifetime of (x) .
2. The p.d.f. of T is $f_T(t) = 2e^{-2t}$, $t \geq 0$.

Calculate $\text{Var}[T]$.

A. 0.25

B. 0.50

C. 1.00

D. 2.00

E. 4.00

100. You are given:

1. T is the random variable for the future lifetime of (x) .
2. The p.d.f. of T is $f_T(t) = 2e^{-2t}$, $t \geq 0$.

Calculate $m(x)$, the median future lifetime of (x) .

A. $\frac{e^{-4}}{2}$

B. $\frac{e^{-2}}{2}$

C. $\frac{\ln 2}{2}$

D. $\frac{\ln 4}{2}$

E. 1

101. You are given:

1. T is the random variable for the future lifetime of (x) .
2. The p.d.f. of T is $f_T(t) = 2e^{-2t}$, $t \geq 0$.

Calculate m_x , the central-death-rate at age x .

A. $\frac{e^{-2}}{2}$

B. e^{-2}

C. $2e^{-2}$

D. 1

E. 2

104. Deaths are uniformly distributed between integral ages.

Which of the following represents ${}_3/4p_x + \frac{1}{2}({}_1/2p_x \mu_{x+1/2})$?

A. ${}_3/4p_x$

B. ${}_3/4q_x$

C. ${}_1/2p_x$

D. ${}_1/2q_x$

E. ${}_1/4p_x$

105. You are given the following survival function:

$$s(x) = \begin{cases} \frac{10,000-x^2}{10,000} & 0 \leq x \leq 100 \\ 0 & x > 100 \end{cases}$$

Calculate q_{32} .

- A. Less than 0.005
- B. At least 0.005, but less than 0.006
- C. At least 0.006, but less than 0.007
- D. At least 0.007, but less than 0.008
- E. At least 0.008

106. You are given:

- A survival distribution is defined by

$$\ell_x = 1000 \left[1 - \left(\frac{x}{100} \right)^2 \right], 0 \leq x \leq 100$$

- μ_x denotes the actual force of mortality for the survival function
- μ_x^L denotes the approximation of the force of mortality based on the uniform distribution of deaths assumption for ℓ_x , $50 \leq x < 51$

Calculate $\mu_{50.25} - \mu_{50.25}^L$.

A. -0.00016

B. -0.00007

C. 0.00000

D. 0.00007

E. 0.00016

107. Which of the following formulas could serve as a force of mortality?

1. $\mu_x = BC^x$, $B > 0, C > 1$
2. $\mu_x = a(b+x)^{-1}$, $a > 0, b > 0$
3. $\mu_x = (1+x)^{-3}$, $x \geq 0$

A. 1 only

B. 2 only

C. 3 only

D. 1 and 2 only

E. 1 and 3 only

109. Based on given values of ℓ_x and ℓ_{x+1} , ${}_1/4p_{x+1/4} = 49/50$ under the assumption of constant force of mortality.

Calculate ${}_1/4p_{x+1/4}$ under the uniform distribution of deaths hypothesis.

A. 0.9799

B. 0.9800

C. 0.9801

D. 0.9802

E. 0.9803

110. Mortality follows Makeham's law, $\mu_x = A + Bc^x$.

Which of the following represents $\int_1^{\infty} {}_t p_x \mu_{x+t} dt$?

A. p_x

B. q_x

C. 1

D. 0

E. μ_x

112. A mortality study is conducted for the age interval $(x, x + 1]$.

If a constant force of mortality applies over the interval, then ${}_{0.25}q_{x+0.1} = 0.05$.

Calculate ${}_{0.25}q_{x+0.1}$ assuming a uniform distribution of deaths applies over the interval.

A. 0.044

B. 0.047

C. 0.050

D. 0.053

E. 0.056

113. Which of the following are true?

1. ${}_t+uq_x \geq {}_uq_{x+t}$ for $t \geq 0$ and $u \geq 0$.

2. ${}_uq_{x+t} \geq {}_t|uq_x$ for $t \geq 0$ and $u \geq 0$.

3. If $s(x)$ follows DeMoivre's law, the median future lifetime of (x) equals the mean future lifetime of (x) .

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. 1, 2 and 3

E. The correct answer not given by (A), (B), (C) or (D)

114. From a population mortality study, you are given:

- Within each age interval, $(x + k, x + k + 1]$, the force of mortality, $\mu(x + k)$, is constant.

k	$e^{-\mu(x+k)}$	$\frac{1 - e^{-\mu(x+k)}}{\mu(x+k)}$
0	0.98	0.99
1	0.96	0.98

Calculate ${}^{\circ}e_{x:\overline{2}|}$, the expected lifetime in years over $(x, x + 2]$.

A. 1.92

B. 1.94

C. 1.95

D. 1.96

E. 1.97

116. You are given the following information:

- $\ell_1 = 9700$
- $q_1 = q_2 = 0.020$
- $q_4 = 0.026$
- $d_3 = 232$

Determine the expected number of survivors to age 5.

- A. Less than 8,845
- B. At least 8,845, but less than 8,850
- C. At least 8,850, but less than 8,855
- D. At least 8,855, but less than 8,860
- E. At least 8,860

117. You are given:

$$s(x) = \frac{(10-x)^2}{100}, 0 \leq x \leq 10$$

Instead of using $s(x)$ within each year of age, use the constant force of mortality assumption to calculate the average number of years lived between ages 1 and 2 by those of the survivorship group who die between those ages.

A. 0.461

B. 0.480

C. 0.490

D. 0.500

E. 0.508

118. You are given:

$$s(x) = \frac{(10-x)^2}{100}, 0 \leq x \leq 10$$

Calculate the difference between the force of mortality at age 1, and the probability that (1) dies before age 2.

A. 0.007

B. 0.010

C. 0.012

D. 0.016

E. 0.024

119. You are given:

$$s(x) = \frac{(10-x)^2}{100}, \quad 0 \leq x \leq 10$$

Calculate the average number of years lived between 1 and 2 by those of the survivorship group who die between those ages.

A. 0.461

B. 0.473

C. 0.484

D. 0.490

E. 0.500

120. You are given:

$$s(x) = \frac{(10-x)^2}{100}, \quad 0 \leq x \leq 10$$

Calculate the average number of years of future lifetime of the ℓ_1 survivors of the group at age 1.

A. 2.37

B. 2.43

C. 2.70

D. 2.92

E. 3.00

121. You are given the following:

- The probability that a person age 20 will survive 30 years is 0.7.
- The probability that a person age 45 will die within 5 years and that another person age 40 will survive 5 years is 0.0475.
- The probability that a person age 20 will survive 20 years and that another person age 40 will die within 5 years is 0.04.

Calculate the probability that a person age 20 will survive 25 years.

A. 0.74

B. 0.75

C. 0.76

D. 0.77

E. 0.78

124. Which of the following statements is true concerning the inequality $e_{x+1} > e_x$?

A. The inequality cannot be true.

B. The inequality is true if and only if: $e_{x+1} > \frac{p_x}{q_{x+1}}$

C. The inequality is true if and only if: $e_{x+1} > \frac{p_x}{p_{x+1}q_{x+1}}$

D. The inequality is true if and only if: $e_{x+1} > \frac{p_x+1}{q_x}$

E. The inequality is true if and only if: $e_{x+1} > \frac{p_x}{q_x}$

126. You are given:

- The force of mortality is a constant μ
- $\mu \leq 1$
- ${}_3|_3q_{33} = 0.0030$

Calculate 1000μ .

A. 0.8

B. 0.9

C. 1.0

D. 1.1

E. 1.2

127. You are given the following mortality table:

x	ℓ_x	q_x	d_x
50	1,000	0.020	
51			32
52			30
53			28
54		0.028	

In a group of 800 people age 50, determine the expected number who will die while age 54.

- A. Less than 21
- B. At least 21, but less than 24
- C. At least 24, but less than 27
- D. At least 27, but less than 30
- E. At least 30

132. Consider a subgroup of lives who have been exposed to a certain disease. It is estimated that this subgroup will have a higher mortality for two years following exposure to this disease. The mortality rate is 10% higher than normal during the first year and 5% higher during the second year. After that the mortality returns to normal.

You are given:

- $q_x = 0.07$
- $q_{x+1} = 0.10$
- $q_{x+2} = 0.11$
- $e_{x+3} = 5$

Calculate the reduction in curtate life expectancy, in years, for a person age (x) who has just been exposed to this disease.

- A. Less than 0.050
- B. At least 0.050, but less than 0.075
- C. At least 0.075, but less than 0.100
- D. At least 0.100, but less than 0.125
- E. At least 0.125

133. In a certain population, the force of mortality is constant.

If the probability that a life age 60 will survive to age 80 is 0.10, what is the force of mortality?

- A. Less than 0.10
- B. At least 0.10, but less than 0.12
- C. At least 0.12, but less than 0.14
- D. At least 0.14, but less than 0.16
- E. At least 0.16

134. You are given the following information:

1. The probability that two 70-year-olds are both alive in 20 years is 16%.
2. The probability that two 80-year-olds are both alive in 20 years is 1%.
3. There is an 8% chance of a 70-year-old living 30 years.
4. All lives are independent and have the same expected mortality.

Determine the probability of an 80-year-old living 10 years.

- A. Less than 0.35
- B. At least 0.35, but less than 0.45
- C. At least 0.45, but less than 0.55
- D. At least 0.55, but less than 0.65
- E. At least 0.65

137. Light bulbs burn out according to the following life table:

x	ℓ_x
0	1,000,000
1	800,000
2	600,000
3	300,000
4	0

A new plant has 2,500 light bulbs. Burned out light bulbs are replaced with new light bulbs at the end of each year.

What is the expected number of new light bulbs that will be needed at the end of year 3?

- A. Less than 800
- B. At least 800, but less than 860
- C. At least 860, but less than 920
- D. At least 920, but less than 980
- E. At least 980

138. You are given:

- Mortality follows DeMoivre's law.
- $\text{Var}[T(50)] = 192$.

Calculate ω .

A. 98

B. 100

C. 107

D. 110

E. 114

139. [3.S00.1] Given:

1. ${}^{\circ}e_0 = 25$
2. $\ell_x = \omega - x, 0 \leq x \leq \omega$
3. $T(x)$ is the future lifetime random variable.

Calculate $\text{Var}[T(10)]$.

A. 65

B. 93

C. 133

D. 178

E. 333

140. [3.S00.12] For a certain mortality table, you are given:

1. $\mu(80.5) = 0.0202$
2. $\mu(81.5) = 0.0408$
3. $\mu(82.5) = 0.0619$
4. Deaths are uniformly distributed between integral ages.

Calculate the probability that a person age 80.5 will die within two years.

A. 0.0782

B. 0.0785

C. 0.0790

D. 0.0796

E. 0.0800

141. [3.S00.17] The future lifetimes of a certain population can be modeled as follows:

- Each individual's future lifetime is exponentially distributed with constant hazard rate θ .
- Over the population, θ is uniformly distributed over $(1,11)$.

Calculate the probability of surviving to time 0.5, for an individual randomly selected at time 0.

A. 0.05

B. 0.06

C. 0.09

D. 0.11

E. 0.12

142. [3.S00.21] A risky investment with a constant rate of default will pay:

- principal and accumulated interest at 16% compounded annually at the end of 20 years if it does not default; and
- zero if it defaults.

A risk-free investment will pay principal and accumulated interest at 10% compounded annually at the end of 20 years.

The principal amounts of the two investments are equal.

The actuarial present value of the two investments are equal at time zero.

Calculate the median time until default or maturity of the risky investment

- A. 9 B. 10 C. 11 D. 12 E. 13

143. [3.S00.28] For a mortality study on college students:

- Students entered the study on their birthdays in 1963.
- You have no information about mortality before birthdays in 1963.
- Dick, who turned 20 in 1963, died between his 32nd and 33rd birthdays.
- Jane, who turned 21 in 1963, was alive on her birthday in 1998, at which time she left the study.
- All lifetimes are independent.
- Likelihoods are based upon the Illustrative Life Table.

Calculate the likelihood for these two students.

A. 0.00138

B. 0.00146

C. 0.00149

D. 0.00156

E. 0.00169

144. [3.F00.4] Mortality for Audra, age 25, follows DeMoivre's law with $\omega = 100$. If she takes up hot air ballooning for the coming year, her assumed mortality will be adjusted so that for the coming year only, she will have a constant force of mortality of 0.1.

Calculate the decrease in the 11-year temporary complete life expectancy for Audra if she takes up hot air ballooning.

- A. 0.10 B. 0.35 C. 0.60 D. 0.80 E. 1.00

146. [3.F00.25] Given:

- Superscripts M and N identify two forces of mortality and the curtate expectations of life calculated from them.
- $\mu_{25}^N(t) = \begin{cases} \mu_{25}^M(t) + 0.1(1 - t) & 0 \leq t \leq 1 \\ \mu_{25}^M(t) & t > 1 \end{cases}$
- $e_{25}^M = 10.0$

Calculate e_{25}^N .

A. 9.2

B. 9.3

C. 9.4

D. 9.5

E. 9.6

147. [3.F00.31] For an industry-wide study of patients admitted to hospitals for treatment of cardiovascular illness in 1998, you are given:

Duration in Days	Number of Patients Remaining Hospitalized
0	4,386,000
5	1,461,554
10	486,739
15	161,801
20	53,488
25	17,384
30	5,349
35	1,337
40	0

Discharges from the hospital are uniformly distributed between the durations shown in the table.

Calculate the mean residual time remaining hospitalized, in days, for a patient who has been hospitalized for 21 days.

- A. 4.4 B. 4.9 C. 5.3 D. 5.8 E. 6.3

148. [3.F00.36] Given:

- $\mu(x) = F + e^{2x}, x \geq 0$
- ${}_{0.4}p_0 = 0.50$

Calculate F .

A. -0.20

B. -0.09

C. 0.00

D. 0.09

E. 0.20

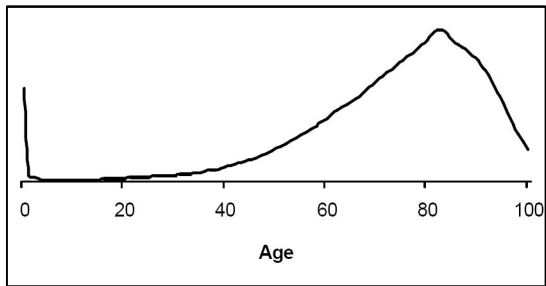
149. [3.S01.1] For a given life age 30, it is estimated that an impact of a medical breakthrough will be an increase of 4 years in ${}^{\circ}e_{30}$, the complete expectation of life.

Prior to the medical breakthrough, $s(x)$ followed DeMoivre's law with $\omega = 100$ as the limiting age.

Assuming DeMoivre's law still applies after the medical breakthrough, calculate the new limiting age.

- A. 104 B. 105 C. 106 D. 107 E. 108

151. [3.S01.14] The following graph is related to current human mortality:



Which of the following functions of age does the graph most likely show?

A. $\mu(x)$

B. $\ell_x\mu(x)$

C. $\ell_x p_x$

D. ℓ_x

E. ℓ_x^2

152. [3.S01.27] An actuary is modeling the mortality of a group of 1000 people, each age 95, for the next three years.

The actuary starts by calculating the expected number of survivors at each integral age by

$$\ell_{95+k} = 1000 {}_k p_{95}, \quad k = 1, 2, 3$$

The actuary subsequently calculates the expected number of survivors at the middle of each year using the assumption that deaths are uniformly distributed over each year of age.

This is the result of the actuary's model:

Age	Survivors
95.0	1000
95.5	800
96.0	600
96.5	480
97.0	—
97.5	288
98.0	—

The actuary decides to change his assumption for mortality at fractional ages to the constant force assumption. He retains his original assumption for each ${}_k p_{95}$.

Calculate the revised expected number of survivors at age 97.5.

- A. 270 B. 273 C. 276 D. 279 E. 282

153. [3.S01.28] For a population of individuals, you are given:

- Each individual has a constant force of mortality.
- The forces of mortality are uniformly distributed over the interval $(0,2)$.

Calculate the probability that an individual drawn at random from this population dies within one year.

A. 0.37

B. 0.43

C. 0.50

D. 0.57

E. 0.63

154. [3.S01.33] For a 4-year college, you are given the following probabilities for dropout from all causes:

- $q_0 = 0.15$
- $q_1 = 0.10$
- $q_2 = 0.05$
- $q_3 = 0.01$

Dropouts are uniformly distributed over each year.

Compute the temporary 1.5-year complete expected college lifetime of a student entering the second year, ${}_1\bar{e}_{1:\overline{1.5}|}$.

- A. 1.25 B. 1.30 C. 1.35 D. 1.40 E. 1.45

155. [3.F01.1] You are given:

$$\mu(x) = \begin{cases} 0.04 & 0 < x < 40 \\ 0.05 & x > 40 \end{cases}$$

Calculate ${}^{\circ}e_{25:\overline{25}|}$.

A. 14.0

B. 14.4

C. 14.8

D. 15.2

E. 15.6

157. [3.F01.37] For watches produced by a certain manufacturer:

- Lifetimes follow a single-parameter Pareto distribution with $\alpha > 1$ and $\theta = 4$.
- The expected lifetime of a watch is 8 years.

You are given the following information about a single-parameter Pareto:

- $f(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$
- $F(x) = 1 - \left(\frac{\theta}{x}\right)^\alpha, x > \theta$
- $E[X] = \frac{\alpha\theta}{\alpha-1}$

Calculate the probability that the lifetime of a watch is at least 6 years.

A. 0.44

B. 0.50

C. 0.56

D. 0.61

E. 0.67

158. [3.F02.1] Given: The survival function $s(x)$, where

$$s(x) = \begin{cases} 1 & 0 \leq x < 1 \\ 1 - \frac{e^x}{100} & 1 \leq x < 4.5 \\ 0 & 4.5 \leq x \end{cases}$$

Calculate $\mu(4)$.

A. 0.45

B. 0.55

C. 0.80

D. 1.00

E. 1.20

159. [3.F02.35] You are given:

- $R = 1 - e^{-\int_0^1 \mu_x(t) dt}$
- $S = 1 - e^{-\int_0^1 (\mu_x(t) + k) dt}$
- k is a constant such that $S = 0.75R$

Determine an expression for k .

A. $\ln [(1 - q_x) / (1 - 0.75q_x)]$

B. $\ln [(1 - 0.75q_x) / (1 - p_x)]$

C. $\ln [(1 - 0.75p_x) / (1 - p_x)]$

D. $\ln [(1 - p_x) / (1 - 0.75q_x)]$

E. $\ln [(1 - 0.75q_x) / (1 - q_x)]$

161. [3-SOA.F03.17] T , the future lifetime of (0) , has a spliced distribution.

- $f_1(t)$ follows the Illustrative Life Table.
- $f_2(t)$ follows DeMoivre's law with $\omega = 100$.
- $f_T(t) = \begin{cases} kf_1(t) & 0 \leq t \leq 50 \\ 1.2f_2(t) & 50 < t \end{cases}$

Calculate ${}_{10}p_{40}$.

A. 0.81

B. 0.85

C. 0.88

D. 0.92

E. 0.96

162. [3-SOA.F03.18] A population has 30% who are smokers with a constant force of mortality 0.2 and 70% who are non-smokers with a constant force of mortality 0.1.

Calculate the 75th percentile of the distribution of the future lifetime of an individual selected at random for this population.

- A. 10.7 B. 11.0 C. 11.2 D. 11.6 E. 11.8

163. [3-SOA.F03.28] For (x) :

- K is the curtate future lifetime random variable.
- $q_{x+k} = 0.1(k+1)$, $k = 0, 1, 2, \dots, 9$

Calculate $\text{Var}(K \wedge 3)$.

A. 1.1

B. 1.2

C. 1.3

D. 1.4

E. 1.5

164. [3-SOA.F03.35] For T , the future lifetime random variable for (0) :

- $\omega > 70$
- ${}_{40}p_0 = 0.6$
- $E(T) = 62$
- $E(T \wedge t) = t - 0.005t^2, 0 < t < 60$

Calculate the complete expectation of life at 40.

A. 30

B. 35

C. 40

D. 45

E. 50

166. [3-CAS.F03.4] Given:

$$\mu(x) = \frac{2}{100 - x}, \text{ for } 0 \leq x < 100$$

Calculate ${}_{10|}q_{65}$.

A. $\frac{1}{25}$

B. $\frac{1}{35}$

C. $\frac{1}{45}$

D. $\frac{1}{55}$

E. $\frac{1}{65}$

167. [3-CAS.F03.5] Given:

- Mortality follows De Moivre's law.
- ${}^{\circ}e_{20} = 30$

Calculate q_{20} .

A. $\frac{1}{60}$

B. $\frac{1}{70}$

C. $\frac{1}{80}$

D. $\frac{1}{90}$

E. $\frac{1}{100}$

168. [3-CAS.S04.10] 4,000 people age (30) each pay an amount, P , into a fund. Immediately after the 1,000th death, the fund will be dissolved and each of the survivors will be paid \$50,000.

- Mortality follows the Illustrative Life Table, using linear interpolation at fractional ages.
- $i = 12\%$

Calculate P .

- A. Less than 515
- B. At least 515, but less than 525
- C. At least 525, but less than 535
- D. At least 535, but less than 545
- E. At least 545

169. [3-SOA.F04.4] For a population which contains equal numbers of males and females at birth:

- For males, $\mu^m(x) = 0.10, x \geq 0$
- For females, $\mu^f(x) = 0.08, x \geq 0$

Calculate q_{60} for this population.

- A. 0.076 B. 0.081 C. 0.086 D. 0.091 E. 0.096

170. [3-SOA.F04.24] The future lifetime of (0) follows a two-parameter Pareto distribution with $\theta = 50$ and $\alpha = 3$.

You are given the following information about the two-parameter Pareto distribution:

- $f(x) = \frac{\alpha\theta^\alpha}{(\theta + x)^{\alpha+1}}$
- $F(x) = 1 - \left(\frac{\theta}{\theta + x}\right)^\alpha$
- $E[X] = \frac{\theta}{\alpha - 1}$

Calculate ${}^{\circ}e_{20}$.

- A. 5 B. 15 C. 25 D. 35 E. 45

171. [3-CAS.F04.8] Given:

$$s(x) = \left[1 - \frac{x}{100}\right]^{\frac{1}{2}}, \text{ for } 0 \leq x \leq 100$$

Calculate the probability that a life age 36 will die between ages 51 and 64.

- A. Less than 0.15
- B. At least 0.15, but less than 0.20
- C. At least 0.20, but less than 0.25
- D. At least 0.25, but less than 0.30
- E. At least 0.30