The Infinite Actuary's

Sample Practice Questions for the

QFI Quant Exam

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Note: This PDF shows sample practice questions that are included in the TIA seminar. The TIA QFI QF seminar includes solutions to 100+ exam-style questions.

1. (16 points = 48 minutes)

It's nice to have a spot where the important stochastic integrals are all together. You need a strong knowledge of stochastic integrals to pass QFI Quant. These are key examples to know. In fact, I recommend doing this question at least five times. These come up over and over again on past exams! It is hard to practice this question "too much", the more you practice it the better!

Throughout this question you may assume that $t > 0$ is a positive constant.

(a) (1 point) Prove that $E(W_t^4) = 3t^2$

(b) (1 point) $\int_0^t W_s dW_s$

(c) (1 point) $\int_0^t dW_s$

(d) (1 point) $\mathbb{E}_0(\int_0^t W_s^2 ds)$

(e) (1 point) $\int_0^t s dW_s + \int_0^t W_s ds$

(f) (1 point) $\int_0^t W_s^2 dW_s + \int_0^t W_s ds$

(g) (2 points) Calculate $\mathbb{V}_0(\int_0^t W_s ds)$

(h) (2 points) Calculate $\mathbb{V}_0(\int_0^t W_s^2 ds)$

(i) (1 point) Calculate $\mathbb{E}_0(\int_0^t (5s^2 + 7\sqrt{2}) W_s dW_s)$

(j) (1 point) Johnny jots down the following work to solve part (g). Is Johnny right?

$$\mathbb{V}_0(\int_0^t W_s ds) = \int_0^t V_0(W_s) ds = \int_0^t s ds = \frac{t^2}{2}$$

(k) (2 points) Jose writes down some scratch work below to solve part (g). Is Jose's approach correct? Explain why or why not.

To find the variance of Int W_s ds, I take the Expectation of (tW_t – int s dW_s)^2 which by FOIL gives me E[t^2 x W^2] –2E[a term that goes to 0] + E[int s^2 ds] = t^3 – 2 * 0 + t^3/3= 4/3 * t^3.

(l) (1 point) Calculate $\mathbb{E}_0[tW_t \int_0^t s dW_s]$

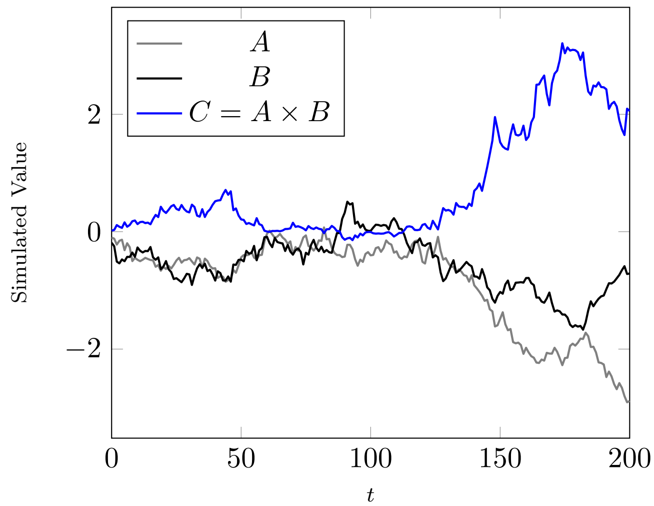
(m) (1 point) Calculate $\mathbb{E}_0(e^{c \int_0^T W_t dt})$

2. (5 points = 15 minutes)

Suppose that $A = W_{1,t}$ and $B = W_{2,t}$ where A and B have correlation equal to ρ.

C is defined as the product of A and B.

A simulated path is shown below:



(a) (1 point) For each value of ρ below, determine $E(C)$ and $V(C)$.

(i) $\rho = 1$

(ii) $\rho = -1$

(iii) A and B are independent, and so $\rho = 0$

C is actually *not* normally distrbuted. In fact, the distribution of C is very complicated. However, it is possible to further understand C through the following change of variables:

$$C = \rho W_{1,t}^2 + \sqrt{1-\rho^2} W_{1,t} Z_t = W_{1,t}\left[\rho W_{1,t} + \sqrt{1-\rho^2} Z_t\right]$$

Here, $Z_t \sim N(0,t)$ is independent of $W_{1,t}$.

(b) (1 point) Determine $E(C)$

(c) (1.5 points) Determine $V(C)$

(d) (1 point) Determine the value(s) of ρ that maximize/minimize the variance of C

(e) (.5 points) Determine $\text{Corr}\left(W_{1,t}, \rho W_{1,t} + \sqrt{1-\rho^2} Z_t\right)$.

3. (7 points = 21 minutes)

Suppose that U, Y and Z are defined as follows:

$$\frac{dY}{Y} = adt + bdW_Y$$

$$\frac{dZ}{Z} = fdt + gdW_Z$$

$$U = YZ$$

The two Weiner processes dW_Y and dW_Z are have correlation ρ.

You are given the initial conditions $Y_0 = Z_0 = 1$.

(a) (1 point) The *stochastic product rule* states that for stochastic processes Y and Z:

$$d(Y \cdot Z) = YdZ + ZdY + (dZ)(dY)$$

Derive the stochastic product rule using multidimensional Ito's Lemma on $f = Y \cdot Z$

(b) (2 points) Determine the Ito process for U

(c) (2 points) Provide an explicit solution for:

(i) Y

(ii) Z

(iii) U

(d) (1 point) What is the distribution of U?

(e) (.5 points) Analyze the following statement:

The product of two correlated GBM processes is GBM.

(f) (.5 points) Analyze the following statement:

If X_t is GBM, then X_t^k is GBM for all integers $k \geq 1$.

Note: To be clear on notation, Z is NOT a standard brownian motion but W is a standard brownian motion.

4. (5 points = 15 minutes)

(a) (2 points) Solve for $r(t)$ given the following SDE:

$$dr(t) = a\left(b - r(t)\right)dt + \sigma dW_t$$

(b) (2 points) You are given that $S = \int_0^t \sigma e^{-a(t-s)} dW_s$ where S is a normally distributed random variable.

(i) Calculate $\mathbb{E}_0(S)$

(ii) Calculate $\mathbb{V}_0(S)$

(c) (1 point) Calculate the following:

(i) $\mathbb{E}_0\left[r(t)\right]$

(ii) $\mathbb{V}_0\left[r(t)\right]$

5. (6 points = 18 minutes)

Parts (a) and (c) of this question require an Excel spreadsheet. For these parts, please open up the corresponding Excel workbook template for this question and perform the necessary calculations. You can then download the solution version of the Excel template for this section to analyze the solutions. We will also provide a detailed solution in this document describing the calculation steps

Insurance Company ABC has recently sold a variable annuity with GMAB rider that contains the following features:

- The initial premium (and thus initial account value) is \$100,000
- The total fee is 1% of account value
- The policyholder is currently aged 65, and the GMAB will reset every three years
- The guarantee base starts at the initial premium, and will ratchet up to the account value (if AV is greater) at the end of every three years

Assume the Black Scholes model is used to perform risk-neutral pricing on this guarantee rider. You are given the following additional capital market assumptions:

- The policyholder's account value is invested in an S&P 500 index fund with an assumed volatility of 18%
- The risk-free rate is 2%, and the fund does not pay dividends

(a) (2 points) Using the product and market information described above, as well as the mortality rates provided in the Excel template, calculate the no-arbitrage value of the gross liability of the GMAB for the first component that will mature in three years

$\boxed{\text{The response for this part is to be provided in the Excel spreadsheet}}$

(b) (1 point) Can the same formula in part (a) be used to calculate the gross liability of the second GMAB component that will mature in six years? Explain your answer.

Now, consider the following equity returns for SPX over the next ten years:

Year	S&P 500 Return
1	4.5%
2	2.5%
3	8%
4	1%
5	3%
6	-12%
7	3%
8	0%
9	-6%
10	15%

(c) (1 point) Calculate what the policyholder account value and guarantee base will be equal to at the end of year 10

The response for this part is to be provided in the Excel spreadsheet

The chief actuary is considering expanding sales of another type of guaranteed rider, a GMWB (guaranteed minimum withdrawal benefit), on the variable annuity. He asks his analyst, Jim, to provide more information on basic features of a GMWB. Jim mentions the following:

- GMWB's are a great product to offer because they will have lower interest rate risk than a typical GMAB
- However, GMWB's may result in higher losses for the insurance company during the first few years of the product, since the policyholder may start taking guaranteed withdrawals during that time
- The insurance company should consider converting the GMWB to a lifetime guaranteed withdrawal rider in order to reduce equity risk
- If the company offers a roll-up option to the guarantee base of the GMWB, then it should charge a higher rider charge for the product

(d) (2 points) Assess whether or not you agree with each of Jim's statements:

