Hurlimann

Tasks:

2. Calculate unpaid claims estimates
3. Test unpaid claims estimates for reasonableness
6. Estimate parameters of unpaid claims distributions
11. Test output of unpaid claim distributions for reasonableness

Hurlimann's credible loss ratio method for estimating reserves is a credibility weighting of the chain ladder and Bornhuetter-Ferguson reserves. Sound familiar? Yes, this is similar to the Benktander method. In fact, we're going to see the Benktander method reappear. The difference is in how we calculate the credibility factor, including the calculation of the percent paid or reported to date.

Advantages:

1. Calculations are straightforward.
2. As long as the same premiums are used, different actuaries will always achieve the same results.
3. Using the optimal credibility weights minimizes both the MSE and the variance of the claims reserve.

Notation

For origin period i reported in period k:

S_{ik} = incremental paid claims

C_{ik} = cumulative paid claims

R_i = reserves

U_i = ultimate claims

V_i = premium

m_k = incremental loss ratio

p_i = loss ratio payout factor (or loss ratio lag factor or percent paid)

q_i = loss ratio reserve factor (percent unpaid)

Assumptions

The following assumptions are necessary in using Hurlimann's method:

- Losses are fully developed at the end of the triangle (i.e. the tail factor is 1.0).
- $Var(U_i) = Var(U_i^{BC})$
  - This is Hurlimann's notation for saying that the variance of the ultimate using his method should equal the variance of the ultimate using the expected loss method (U_i^{BC} = ELR x premium).
  - When you see this assumption in an exam question, it should be a clue that you need to use Hurlimann's method.
- $E(U_i) = E(U_i^{BC})$, which is just a fancy way of saying that the ultimate should be unbiased.

Calculating Hurlimann Ultimate Losses

Example: You are given the following incremental loss development triangle and earned premiums for a book of business. Calculate the Benktander, Neuhaus, and Optimal Credibility reserves for each year. Assume $Var(U_i) = Var(U_i^{BC})$.

	1	2	3	4	Premium
2020	5,525	2,500	875	475	10,250
2021	4,500	2,600	900		9,900
2022	4,750	2,560			10,000
2023	4,900				9,500

Step 1: Calculate m, the incremental loss ratio.

$$m_k = \frac{E\left[\sum_{i=1}^{n-k+1} S_{ik}\right]}{\sum_{i=1}^{n-k+1} V_i}, \qquad k = 1, ..., n$$

$$\text{incremental loss ratio} = \frac{\text{sum of incremental losses in a reporting period}}{\text{sum of the corresponding premiums}}$$

$$m_1 = \frac{5{,}525 + 4{,}500 + 4{,}750 + 4{,}900}{10{,}250 + 9{,}900 + 10{,}000 + 9{,}500} = 0.49622$$

$$m_2 = \frac{2{,}500 + 2{,}600 + 2{,}560}{10{,}250 + 9{,}900 + 10{,}000} = 0.25406$$

$$m_3 = \frac{875 + 900}{10{,}250 + 9{,}900} = 0.08809$$

$$m_4 = \frac{475}{10{,}250} = 0.04634$$

We can also calculate the total loss ratio, which we will use in future steps, as

$$LR = \sum_{i=1}^{n} m_k = 0.49622 + 0.25406 + 0.08809 + 0.04634 = 0.88471$$

Step 2: Calculate p and q, the percent reported and the percent unreported.

$$p_i = \frac{\sum_{k=1}^{n-i+1} m_k}{\sum_{k=1}^{n} m_k}, \qquad i = 1, ..., n$$

$$\text{percent reported} = \frac{\text{sum of incremental loss ratios for reported periods}}{\text{total loss ratio}}$$

$$p_{2020} = \frac{0.49622 + 0.25406 + 0.08809 + 0.04634}{0.88471} = 1.0$$

$$p_{2021} = \frac{0.49622 + 0.25406 + 0.08809}{0.88471} = 0.94762$$

$$p_{2022} = \frac{0.49622 + 0.25406}{0.88471} = 0.84805$$

$$p_{2023} = \frac{0.49622}{0.88471} = 0.56088$$

$$q_i = 1 - p_i = \text{percent unreported}$$

$$q_{2020} = 0.0$$

$$q_{2021} = 0.05238$$

$$q_{2022} = 0.15195$$

$$q_{2023} = 0.43912$$

Step 3: Calculate the chain ladder reserves, also known as the individual reserves.

Note that Hurlimann calls these the individual reserves because they depend solely on loss experience from an individual *origin period.*

$$R^{ind} = \frac{q_i}{p_i} C_{i,n-i+1}, \qquad i = 1, ..., n$$

Reserves $= \frac{\text{percent unreported}}{\text{percent reported}}$ x cumulative losses reported

AY	p	q	C	R^{ind}
2020	1.00000	0.00000	9,375	$\frac{0.00000}{1.00000} \times 9{,}375 = 0$
2021	0.94762	0.05238	8,000	$\frac{0.05238}{0.94762} \times 8{,}000 = 442$
2022	0.84805	0.15195	7,310	$\frac{0.15195}{0.84805} \times 7{,}310 = 1{,}310$
2023	0.56088	0.43912	4,900	$\frac{0.43912}{0.56088} \times 4{,}900 = 3{,}836$

Step 4: Calculate the Bornhuetter-Ferguson reserves, also known as the collective reserves.

Note that Hurlimann calls these the collective reserves because they depend on the loss experience of the entire portfolio over all origin periods (i.e. the loss ratio is calculated using experience from all periods).

$$R^{coll} = q_i V_i \sum_{k=1}^{n} m_k, \qquad i = 1, ..., n$$

Reserves = percent unreported x premium x total loss ratio

AY	q	V	R^{coll}
2020	0.00000	10,250	$0.00000 \times 10{,}250 \times 0.88471 = 0$
2021	0.05238	9,900	$0.05238 \times 9{,}000 \times 0.88471 = 459$
2022	0.15195	10,000	$0.15195 \times 10{,}000 \times 0.88471 = 1{,}344$
2023	0.43912	9,500	$0.43912 \times 9{,}500 \times 0.88471 = 3{,}691$

Step 5: Find Z, the credibility factor. We have 3 different options:

a Gunnar Benktander

$Z_i^{GB} = p_i$

b Walter Neuhaus

$Z_i^{WN} = p_i \sum_{k=1}^{n} m_k, \qquad i = 1, ..., n$

c Optimal Credibility

$Z_i^{Opt} = \frac{p_i}{p_i + t_i^*} \qquad \text{where } t_i^* = \frac{f_i - 1 + \sqrt{(f_i + 1)(f_i - 1 + 2p_i)}}{2}, \qquad i = 1, ..., n$

Under the assumption that $Var(U_i) = Var(U_i^{BC})$, $f_i = 1 \Rightarrow t_i^* = \sqrt{p_i}$, and $Z_i^{Opt} = \frac{p_i}{p_i + \sqrt{p_i}}$

AY	p	Z^{GB}	Z^{WN}	Z^{Opt}
2020	1.00000	1.00000	$1.00000 \times 0.88471 = 0.88471$	$\frac{1.00000}{1.00000 + \sqrt{1.00000}} = 0.50000$
2021	0.94762	0.94762	$0.94762 \times 0.88471 = 0.83837$	$\frac{0.94762}{0.94762 + \sqrt{0.94762}} = 0.49328$
2022	0.84805	0.84805	$0.84805 \times 0.88471 = 0.75028$	$\frac{0.84805}{0.84805 + \sqrt{0.84805}} = 0.47941$
2023	0.56088	0.56088	$0.56088 \times 0.88471 = 0.49622$	$\frac{0.56088}{0.56088 + \sqrt{0.56088}} = 0.42822$

Notice that when $f_i = 1$, $Z^{Opt} \leq 0.5$. Z_i will typically be higher for Benktander and Neuhaus than for the optimal credibility method.

Step 6: Calculate the credibility weighted reserves.

$R^c = Z \times R^{ind} + (1 - Z) \times R^{coll}$

Credible Loss Ratio Reserves = Z x Chain Ladder Reserves + (1 - Z) x Bornhuetter-Ferguson Reserves

AY	R^{ind}	R^{coll}	R^{GB}	R^{WN}	R^{Opt}
2020	0	0	0	0	0
2021	442	459	443	445	451
2022	1,310	1,344	1,315	1,318	1,328
2023	3,836	3,691	3,772	3,763	3,753

Mean Squared Error

An estimate of reserves isn't very useful unless we have some confidence that it will be close to the actual future losses. The mean squared error allows us to have some confidence in our estimates. The mean squared error = (actual − expected)^2. A small MSE means that our estimate is close to the actual future losses. The MSEs of the reserves can be calculated generally as follows:

$$mse(R_i) = E[\alpha_i^2(U_i)]\left(\frac{Z_i^2}{p_i} + \frac{1}{q_i} + \frac{(1-Z_i)^2}{t_i}\right)q_i^2$$

If we want to calculate the MSE of the individual reserves, we set $Z = 1$, which allows us to simplify the MSE formula to the following:

$$mse(R_i^{ind}) = E[\alpha_i^2(U_i)]\frac{q_i}{p_i}$$

If we want to calculate the MSE of the collective reserves, we set $Z = 0$, which allows us to simplify the MSE formula as follows:

$$mse(R_i^{coll}) = E[\alpha_i^2(U_i)]q_i\left(1 + \frac{q_i}{t_i}\right)$$

$E[\alpha_i^2(U_i)]$ comes from a distribution of ultimate losses and would be given to you on the exam.

Example: Using the same information given in the prior example, $t_i = \sqrt{p_i}$, and $E[\alpha_i^2(U_i)] = 1{,}000$, calculate the MSE for the individual, collective, and optimal credibility reserves for 2023.

From the prior example, we have the following:

$$p_{2023} = 0.56088$$

$$q_{2023} = 0.43912$$

$$Z^{Opt} = 0.42822$$

From that, we can calculate

$$mse(R_i^{ind}) = 1{,}000 \times \frac{0.43912}{0.56088} = 783$$

$$mse(R_i^{coll}) = 1{,}000 \times 0.43912 \times \left(1 + \frac{0.43912}{\sqrt{0.56088}}\right) = 697$$

$$mse(R_i^{opt}) = 1{,}000 \times \left(\frac{0.42822^2}{0.56088} + \frac{1}{0.43912} + \frac{(1-0.42822)^2}{\sqrt{0.56088}}\right) \times 0.43912^2 = 586$$

Notice that the optimal credibility method has the lowest MSE. It always will. The value of f_i in the optimal credibility weights is calculated in such a way that the MSE of those reserves will always be minimized.

Hurlimann tells us the MSEs for the Benktander and Neuhaus credibility weights will usually be lower than either the individual or collective MSEs, but higher than the optimal credibility MSE. Interestingly enough, though, they aren't much higher than the MSE for the optimal credibility method. This tells us that if it's not possible to calculate the optimal credibility weights, either the Benktander or Neuhaus methods are good alternatives.

Problem Knowledge Checklist

1. Be able to recognize when to use Hurlimann's method
2. Be able to calculate reserves using Hurlimann's method
   - Calculate incremental and total loss ratios
   - Calculate the percent paid/reported
   - Calculate the individual (chain ladder) and collective (BF) reserves
   - Know the three different options for the credibility factor and be able to calculate Z
     - Benktander
     - Neuhaus
     - Optimal Credibility
   - Calculate credibility weighted reserves
3. Be able to calculate the mean squared error (MSE) for the individual, collective, and optimal credibility reserves
4. Be able to interpret the MSE
5. Be able to state the advantages and disadvantages of Hurlimann's method